Andrei Migunov - Research Statement

Previous Research Experience

Thus far, my research has been conducted during my Ph.D. program, as a student in the Laboratory for Molecular Programming (LAMP) at Iowa State University, and under the supervision of my advisor Jack Lutz. I have focused on the theory of algorithmic randomness and the theory of algorithmic dimension which refines randomness from a quality of a mathematical object into a quantity. I also focus on the development of similar randomness notions in the world of molecular programming and analog computation.

In [3], joint work with Xiang Huang and Jack Lutz, we develop a theory of randomness for continuous time and discrete state Markov chains (CTMCs), of which stochastic chemical reaction networks (a programming language structured in terms of reactions between abstract chemical species) are a special case. My current research focuses on a measure of information content for similar (but deterministic) analog computers definable in terms of polynomial ordinary differential equations, similar to the role of Kolmogorov complexity in the classical (digital, Turing) world. I am also working, jointly with Jack Lutz, Xiang Huang, and Elvira Mayordomo, on extending [3] to a theory of dimensions of CTMC trajectories. Lastly, and jointly with Jack Lutz, I am working to establish a new characterization of algorithmic dimension in terms of computable learning functions.

Research Goals

One of my research goals is to contribute to the revitalization of the field of **analog computation** [2]. There are many means to this end, including the recent work of Bournez and Pouly on a universal ordinary differential equation (**ODE**) which can 'simulate' any continuous function whatsoever, given the right parameter [7]. This prompts questions such as "what is **universality** when one computes directly with real numbers rather than with their representations?" and "how does one measure **complexity in analog computation** [1]?". Many of the most well-known results in the digital universe have yet to find analogues in the analog universe. Continuous, rather than discrete, computation could hold many secrets that could help solve existing problems in discrete computational theory, possibly even the P vs NP problem.

Another research goal relates to the ability of computational methods to solve longstanding open problems in classical mathematics which are not about computation. Recent work by Jack Lutz and Neil Lutz established the **Point-to-Set Principle** [4], which has been used to reason about classical **fractal dimensions**[5] using computational tools. I would like to continue in this vein of research and extend this work to the analog universe in terms of continuous computations rather than discrete ones. An important first step would be to clearly define a notion of a **continuous oracle appropriate to ODE computation**, and to establish its relationship to a universal ODE.

Recent work by Zaffora Blando [9] brought me into contact with **computational learning theory** for the first time, and one of my current research projects deals directly with this work. Importantly, learning functions are already known to characterize randomness. However, there are million ways to extend the notion of a learning function to accommodate the millions of intuitions that exist about what it is to learn, and then to investigate the consequences of those conceptual extensions to randomness, or complexity more broadly. Even the theory of childhood development could play a role here if one wanted to employ it. Such connections are waiting to be made, especially at a moment in history when it isn't unusual for a person to think there is a **relationship between the human mind and computation**.

Research Methodology and Philosophy

My approach to my broader area (theoretical computer science) is above all to do what interests me, personally, and in doing so I tend to find problems that are both challenging and important. The process of working on a problem is a quasi-empirical one, and I enjoy that aspect of research. I'm grateful not to be an automated theorem prover because I get to use my intuitions and hunches to make decisions about the angle from which I tackle a problem, which keeps the whole endeavor refreshing and exciting. I also strongly prefer to work on problems which are co-extensive with a practical application. The theory of algorithmic dimension, though interesting in its own right despite not being *applied research* per se, has many immediate applications including establishing relationships between neural connectivity and aging [8]. It's *clear* why it's *important* to develop this theoretical tool, and that's an important aspect of my decision to work on it.

I think that *what mathematics is* is the real practical activity of a community of people, which tends to result in one of many ways of identifying structure in reality. I do think that we discover real things about the world by doing mathematics, and we do so by way of a precise and intimate shared mathematical language. What's fun about working with other people is finding ambiguity and negotiating the terms of that shared language. It's fun to *convince* someone that an approach is useful, and it's fun to *be* convinced by them.

I would also like to mention interdisciplinary work. I feel strongly that interdisciplinary work is not merely the application of the tools and methods of one field to the problems of another. True interdisciplinarity is in the relationships of the categories that two different fields use. It's the interface of two specialist worldviews, rather than the subordination of one to another. 'Analytic' philosophy has maintained a certain engagement with mathematics over the centuries, but so-called 'continental' philosophy and computer science do not tend to have the same relationship. Part of my broader methodology has been to employ notions from this alternate philosophical tradition in my thinking about computation. As an example, thinking of G.W.F. Hegel's writings on logic as prefiguring complexity theory (Hegel is presently being thought about by some in category theory [6]), and of complexity theory as a vehicle for thinking about some of the much broader, even historical problems that Hegel discusses, as an *application* of complexity theory and a means of interrogating its categories.

I have many interests and love nothing more than to see curiosities become entire research programs as they develop a sense of direction and clarity of purpose. Both graduate and undergraduate students have insights of their own that they bring into their research and education, and they should get a chance to follow those interests. To that end, I would welcome a student who wants to bring me into a research program of their own design as much as I want to bring them into mine. I'm willing to leave my scientific comfort zone (within the bounds of my technical abilities) to help foster the research abilities of any graduate students I supervise just as much as I'm willing to leave the comfort of a familiar research area at my own discretion. I would also enjoy mentoring undergraduate students who are interested in theory for the sake of curiosity, and would enthusiastically help them approach original research if they are interested.

References

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